

# POLARISATION-ABERRATION RETRIEVAL FOR HIGH-NA SYSTEMS USING THE EXTENDED NIJBOER-ZERNIKE DIFFRACTION THEORY

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## ABSTRACT

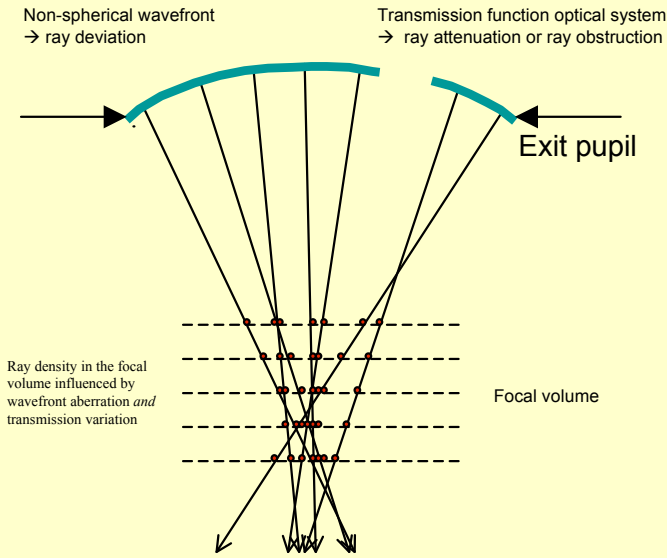
We have derived analytical expressions for the field components in the focal region of a high-numerical-aperture imaging system using the so-called Extended Nijboer-Zernike diffraction theory. It is shown that the transmission function, aberrations and polarisation properties of an imaging system with high numerical aperture can be derived from the through-focus intensity map via an inversion process based on this analysis.

### Problem definition:

How to retrieve optical system properties (*amplitude, phase and polarisation* in the exit pupil) from *intensity* measurements through the *focal volume*?

### 1) Intuitive picture, based on ray optics →

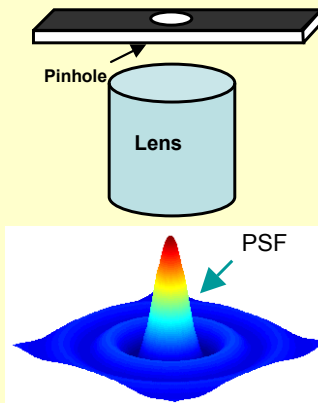
**change in ray direction (wavefront aberration) and ray attenuation determine ray density (intensity) in focal volume !**



### 2) More rigorous picture is based on scalar diffraction theory →

**Huygens-Fresnel diffraction integral for light propagation from exit pupil → image plane.**

**In the presence of aberrations: theory of Nijboer-Zernike (1942)**



**Extension :**

**from source →**

**to exit pupil →**

**to focal volume :**

**Extended Nijboer-Zernike theory for through-focus point-spread function**

For the Bessel series solution, see:

A.J.E.M. Janssen, J. Opt. Soc. Am **A19**, 849-857 (2002)

Basic diffraction integral with defocus: 
$$U(r, \varphi; f) = \frac{1}{\pi} \int_0^1 \rho \exp(ik\rho^2) \int_0^{2\pi} A(\rho, \vartheta) \exp\{i\Phi(\rho, \vartheta)\} \exp\{i2\pi r \rho \cos(\vartheta - \varphi)\} d\vartheta d\rho$$

Introduction of Zernike polynomial expansion representing **amplitude and phase**: 
$$A(\rho, \vartheta) \exp\{i\Phi(\rho, \vartheta)\} = \sum_{n,m} \beta_n^m R_n^{(m)} \exp(im\vartheta)$$

Bessel series solution: 
$$U(r, \varphi; f) = 2 \sum_{n,m} i^m \beta_n^m V_n^{(m)}(r, f) \exp(im\varphi) \quad \text{with:} \quad V_n^{(m)}(r; f) = \exp(ikf) \sum_{l=0}^{\infty} \left(\frac{-if}{\pi r}\right)^l \sum_{j=0}^l u_{lj} \frac{J_{|m|+2j+l+1}(2\pi r)}{2\pi r}$$

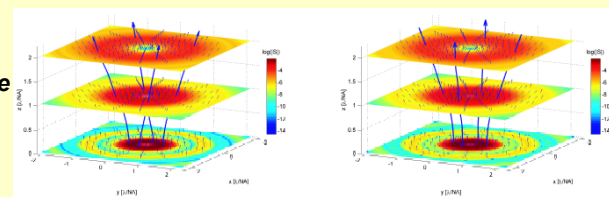
### 3) Vector diffraction theory for high-NA focused beams (forward) →

**ENZ-theory for complex exit pupil function with due account of**

**a) radiometric effect b) high-NA defocusing factor c) polarisation state**

Electric field density in the focal volume is given by 
$$w_E = \frac{\epsilon_0}{4} n^2 |E|^2 = \frac{\epsilon_0}{4} n^2 \{E_1 E_1^* + E_2 E_2^* + E_3 E_3^*\}$$
 with explicit analytic expressions available for electric field vector components in the focal volume.

See: J.J.M. Braat, P. Dirksen, A.J.E.M. Janssen, A.S. van de Nes, J. Opt. Soc. Am. **A20**, 2281-2292 (2003)



Vectorial **forward** calculation using the Extended Nijboer-Zernike theory. Energy flow (arrows) and intensity distribution (colour-coded) in the focal region. Left-hand figure: circularly polarised Right-hand figure: circularly polarised + orbital angular momentum

### 4) Polarisation-aberration retrieval using ENZ-theory at high NA:

**'backward' calculation from energy density in focal volume leads to**

**→ complex lens function + polarisation effects (birefringence)**

State of polarisation in the exit pupil depends on :

- a) geometrical lens properties (NA, transmission, aberration),
- b) birefringence ('scrambling' of polarisation state)

Description of state of polarisation via Jones matrix :

$$\begin{pmatrix} E_{x,j} \\ E_{y,j} \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} a_j \\ b_j \end{pmatrix} \quad \text{with} \quad \begin{pmatrix} a_j \\ b_j \end{pmatrix} \text{ the incident polarisation.}$$

For a pure phase birefringence, the Jones matrix reduces to :

$$\begin{pmatrix} E_{x,j} \\ E_{y,j} \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ -m_{12}^* & m_{11}^* \end{pmatrix} \begin{pmatrix} a_j \\ b_j \end{pmatrix} \quad \text{with} \quad |m_{11}|^2 + |m_{12}|^2 = 1$$

### Conclusion:

**four 'inverse' operations are sufficient for retrieval of the 'polarisation-aberrations' of an optical imaging system**

Detailed information about the ENZ-theory and its applications in optical aberration theory, lithography and lens metrology can be found at the website:

<http://www.nijboerzernike.nl>